647510 **MSC2MatC204x** **Seat No :_____**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
March/April - 2025 (NEW COURSE)
Methods in Partial Differential Equations (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que1(A) Answer any two of the following: (10)

1. Solve the partial differential equation $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$.
   Show that a necessary and sufficient condition that the Pfaffian differential equation $\vec{X} \cdot \overrightarrow{dr} = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$ be integrable is that $\vec{X} \cdot curl\vec{X} = 0$.
2. Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = 0$.

Que-1(B) Answer any one of the following: (04)

1. Solve the partial differential equation $\frac{dx}{y^2+z^2} = \frac{dy}{-xy} = \frac{dz}{-xz}$.
2. Find the integral of the differential equation $yzdx + 2xzdy - 3xydz = 0$.

Oue-2(A) Answer any two of the following: (10)

1. Find the equation of the integral surface of the partial differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$ which passes through the circle $z = 0, x^2 + y^2 = 2x$.
2. Find the equation of the system of surfaces that cut orthogonally the cones of the system $x^2 + y^2 + z^2 = cxy$.
3. If u is a function of x, y and z which satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z} = 0$. Show that u contains x, y and z only in combinations $x + y + z$ and $x^2 + y^2 + z^2$.

Que-2(B) Answer any one of the following: (04)

1. Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.
2. Form a partial differential equation by eliminating the arbitrary functions f and g from $z = yf(x) + xg(y)$.

Que-3(A) Answer any two of the following: (10)

1. Find a complete integral of the partial differential equation $z^2(p^2z^2 + q^2) = 1$.
2. Solve the partial differential equation $p^2x + q^2y = z$ by Jacobi's method.
3. Find the complete integral of the partial differential equation $xp + 3yq = 2(z - x^2q^2)$.

Que-3(B) Answer any one of the following: (04)

1. Find the complete integral of the partial differential equation $z = p^2 - q^2$.
2. Find the complete integral of the partial differential equation $z(p^2 - q^2) = x - y$.

Que-4(A) Answer any two of the following: (10)

1. Solve the partial differential equation $(D^2 - DD' - 6D'^2)z = \cos(2x + y)$.
2. Solve the partial differential equation $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$.
3. Suppose $\beta_\gamma D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ. If $\beta_\gamma \neq 0$, then show that $u_r = \exp\left(-\frac{-\gamma_r y}{\beta_r}\right)\phi_r(\beta_r x)$ is a solution of the equation $F(D, D')z = 0$.

Que-4(B) Answer any one of the following: (04)

1. Solve the partial differential equation $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} = 0$
2. Solve the partial differential equation $(x^2D^2 + 2xyDD' + y^2D'^2)z = x^2y^2$.

Que-5(A) Answer any two of the following: (10)

1. Solve the partial differential equation $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$.
2. Solve the partial differential equation $r + s - 2t = (2x + y)^{\frac{1}{2}}$.
3. Solve the partial differential equation $x^2r - y^2t + px - qy = \log x$.

Que-5(B) Answer any one of the following: (04)

1. Solve the partial differential equation $p^2z + q^2 = 4$.
2. Using Charpit's method, find the complete integral of the partial differential equation $pq = px + qy$.

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